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Efficient Multiple Model Adaptive Estimation in Ballistic Missile Interception Scenarios

Tal Shima,* Yaakov Oshman,† and Josef Shinar‡
Technion—Israel Institute of Technology, 32000 Haifa, Israel

A novel efficient algorithm, featuring a highly reduced computational load, is presented for multiple model adaptive estimation in a future real-life ballistic missile defense scenario, where the blind incoming target (having no information on the interceptor's state) performs a bang–bang evasive maneuver characterized by a random switching time. The efficiency of the algorithm derives mainly from its exploitation of the special structure of the hypothesis space in this problem to drastically reduce the number of concurrently active filters in the bank without incurring any significant performance degradation. The proposed algorithm's efficiency allows a substantial increase in the resolution of the discretized hypothesis space, thus enhancing considerably the attainable estimation performance. The effect of the new estimator's performance on guidance accuracy is examined. The homing performance of various perfect information guidance laws using this efficient estimation method is compared, via Monte Carlo simulations, to the use of a Kalman filter incorporating a shaping filter representing the random target maneuver. The results demonstrate the superiority and viability of the proposed method.

Introduction

GUIDANCE laws of currently used interceptor missiles have commonly been developed based on the assumption of perfect information, that is, ideal noiseless measurements and known inputs to the dynamic system. This assumption has allowed an easier mathematical analysis and, in some cases, even a closed-form solution. It has been of common practice to implement such guidance laws by assuming the well-known certainty equivalence principle[1] (CEP) that states that the optimal control law for a stochastic control problem is the optimal control law for the associated deterministic (certainty equivalent) problem. The validity of the CEP was proved for linear optimal control problems with unbounded control, quadratic cost function, and Gaussian noise (termed linear quadratic Gaussian), with a strictly classical information pattern, where the controller has available all past outputs and controls at any time. For such problems, a well-known separation result states that the estimator and the controller can be designed independently; hence, the estimated states should be used in the control law of the deterministic problem. For problems with a strictly classical information pattern, the state estimator can be designed independently of the control law even if the CEP does not hold. However, the stochastic optimal control law is defined on the space of conditional probability distributions resulting from the solution of the filtering problem, which means that the filtering problem has to be addressed first.[1]

The CEP has never been proved for realistic missile guidance problems, characterized by bounded control, non-Gaussian random target maneuvers, and saturated state variables. Nevertheless, it has been common practice in the guided missile community to assume it when designing the interceptor's control law. In such implementations, target maneuvers were either neglected or assumed to be of a well-defined, mostly constant, structure.[2] Inherently, homing accuracy is limited by the accuracy of the estimated state variables. In guidance laws that explicitly use the target maneuver, the estimation of this variable, which cannot be measured directly, becomes crucial. If the mathematical model (including the target maneuver dynamics) used in the estimator design is inaccurate, the estimation errors become large, leading to a poor homing performance. Previous studies, investigating realistic missile guidance problems with noisy measurements and unknown target maneuvers, concentrated on identifying the actual target maneuver[3] or on searching for an estimator that minimizes the resulting guidance errors in some sense.[4] Future ballistic missile defense (BMD) scenarios against maneuvering reentry vehicles represent a new example for such a case.

Currently known tactical ballistic missiles (TBMs) are not designed to maneuver. Nevertheless, they have an inherent high maneuvering potential in the atmosphere, resulting from their very high reentry speed. Recently developed antiballistic missile defense systems, such as PAC-3 and Arrow, demonstrated the ability to intercept such nonmaneuvering targets with a hit-to-kill accuracy.[5,6] The successful development of such BMD systems is expected to motivate the development of a new generation of maneuvering TBMs in the foreseeable future. Although a TBM is blind with respect to the interceptor, it can execute hard maneuvers randomly (to avoid interception) on its way to a designated surface target, while complying with the constraint of hitting it. If the defense strategy, including the interceptor's guidance law, cannot guarantee that the miss distance generated by an optimal (in the deterministic sense) evasive maneuver is sufficiently small, the probability of an unacceptable leakage does not vanish.

A future BMD scenario against a randomly maneuvering TBM can be analyzed as an imperfect information zero-sum pursuit–evasion game. Such an analysis, based on a simplified mathematical model (linearized kinematics, constant speeds, and low-order system dynamics) has been performed in the past decade.[7–9] In the first phase, perfect information for the interceptor was assumed, but later the reality of noise-corrupted measurements and an estimator were included. The perfect information guidance laws used in these studies were derived using a linear differential game formulation with bounded control[10–12] and the estimator design was based on a standard Kalman filter (KF) with a shaping filter (SF) representing the random target maneuver.[13]

Other estimation methods such as the multiple model adaptive estimator[14] (MMAE) can also be used in a random evasion

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*Ph.D. Student, Department of Aerospace Engineering; currently System Engineer, RAFAEL, Ministry of Defense, Department 35, P.O.B. 2250, 31021 Haifa, Israel; talsh@rafael.co.il. Member AIAA.

†Associate Professor, Department of Aerospace Engineering; Yaakov.Oshman@technion.ac.il. Associate Fellow AIAA.

‡Professor Emeritus, Max & Lottie Dresher Chair of Aerospace Performance and Propulsion, Department of Aerospace Engineering. Fellow AIAA.

encounter[3,15] if the unknown evasion strategy belongs to a finite set of applicable strategies. In the MMAE approach, a set of KFs is run in parallel, each filter corresponding to a different evasion strategy. The estimated state vectors from the various KFs are then fused according to either the minimum mean square error (MMSE) criterion[16] or the maximum a posteriori (MAP) criterion.[17]

In practice, the MMAE technique is seldom applied to interception scenarios because of its associated prohibitive computational load due to the use of a large number of KFs corresponding to the number of assumed evasion strategies. Efficient algorithms have been proposed in the past in the context of maneuvering target tracking for alleviating the computational load problem. In Ref. 18, a multiple model estimator was devised for tracking a maneuvering target, modeling the maneuver via a combination of Singer's exponentially correlated acceleration (ECA) model[19] and the semi-Markov model of Gholson and Moose.[20] Because the models used in Ref. 18 differ only in their inputs, it was observed that the filters' gain and covariance need be computed just once (as opposed to n separate computations, corresponding to the n elemental KFs in the bank). A similar conclusion (although based on some simplifying assumptions) was reached in Ref. 21, which also presented an efficient MMAE for passive tracking of a maneuvering underwater target in the presence of randomly switching biased measurements. In Ref. 22, a bank of extended KF was used to detect target maneuvers, modeled via jumps at random times of an unknown bias that was added to the basic ECA model, within the framework of generalized likelihood ratio filtering. Because this method involved an ever increasing filter bank (because a new filter had to be initialized at every measurement update), the filter bank was arbitrarily trimmed by dropping each filter after it had lived a predetermined number of measurement updates. Yet another computational load alleviation was introduced by Chan et al. in their multiple hypothesis tracking algorithm,[23] which was further developed by Bogler.[24] The adaptive algorithm presented in Refs. 23 and 24 makes use of the fact that, in tracking a maneuvering target modeled via its command input, the only difference between the various hypotheses is the forcing function and its time of application. When this observation was exploited to reduce the computational load, an estimation algorithm equivalent to a MMAE was developed, which consists of one nominal KF (that assumes no target maneuver) and a bank of correction computations that effectively generate the other elemental filters' state estimates and residuals from those computed by the nominal filter. The current work's approach differs from that taken in Refs. 23 and 24; however, here, too, the forcing function's role in distinguishing between the elemental filters is used.

This paper presents a novel efficient algorithm for MMAE, featuring a highly reduced computational load. Moreover, the feasibility and merits of this estimation algorithm are examined in future endoatmospheric BMD scenarios. The efficiency of the algorithm presented derives mainly from its exploitation of the special structure of the hypothesis space in this problem to reduce drastically the number of concurrently active filters in the bank. This order reduction scheme does not incur any significant performance degradation. In addition, when the properties of the dynamic models involved are exploited, the proposed algorithm features one-time covariance and gain computations for all filters in the bank (in a manner similar to the algorithm of Ref. 18). Under realistic computational load constraints, these savings allow a substantial increase in the resolution of the discretized hypothesis space, thus enhancing considerably the attainable estimation performance.

Motivated by that complete separation of estimation and control cannot be asserted in the scenario of interest, the approach taken in this paper emphasizes the estimator's effects on guidance accuracy. Therefore, the performance index used here is based on the miss distance, similar to Refs. 3 and 4. This approach differs from most investigations concerning estimator design, which use estimation accuracy as the figure of merit (implicitly assuming complete separation between estimation and control).

The remainder of this paper is organized as follows. In the following section, the problem of intercepting a maneuvering TBM is formulated. Next, the perfect information maneuver strategies are presented along with an imperfect information analysis. This is followed by a description of the efficient MMAE approach that substantially reduces the computational effort. Then a Monte Carlo simulation study is presented, comparing the merits of the MMAE approach to those of a KF coupled with a SF, for various guidance laws. Concluding remarks are offered in the last section.

Problem Formulation

Assumptions

The investigation of the terminal phase of an endoatmospheric TBM interception scenario is based on the following set of assumptions:

1) The near head-on engagement between the interceptor (pursuer) and TBM (evader) takes place in a plane.

2) Both missiles can be represented by point-mass models with linear control dynamics.

3) The relative endgame trajectory is linearized about a fixed reference line (the initial line of sight or nominal trajectory).

4) Both missiles have constant speeds.

5) The lateral accelerations of both missiles have constant bounds.

6) The maneuvering dynamics of the interceptor and TBM missiles can be approximated by first-order transfer functions with time constants τ_P and τ_E, respectively.

7) The TBM has no information on the state of the interceptor.

8) The interceptor has noisy measurements of some state variables of the engagement.

Dynamic Model

Figure 1 shows a schematic view of the planar engagement geometry. Note that in a near head-on engagement the respective velocity vectors of the missiles are generally not aligned with the reference line of sight (LOS). The aspect angles ϕ_P and ϕ_E are, however, small in an antiballistic interception scenario. Thus, the approximations $\cos(\phi_i) \approx 1$ and $\sin(\phi_i) \approx \phi_i$, $i = P, E$, are uniformly valid and coherent with assumption 3. Moreover, based on assumptions 3 and 4, the final time of the interception can be computed for any given initial conditions of the endgame by

$$t_f = r_0 / V_c \tag{1}$$

where r_0 is the initial range and the closing speed V_c can be closely approximated by the speed sum $V_P + V_E$. The time-to-go is defined as

$$t_{go} = t_f - t \tag{2}$$

and its normalized version is

$$\theta \triangleq t_{go}/\tau_P \tag{3}$$

Remark: In a realistic ballistic missile interception scenario, the range r and range rate $\dot{r} = -V_c$ are continuously, quite accurately, measured by a ground-based radar. This allows uplinking the updated time-to-go to the interceptor missile.

The state vector in the equations of relative motion normal to the reference line is

$$\boldsymbol{X} = [x_1, x_2, x_3, x_4]^T \tag{4}$$

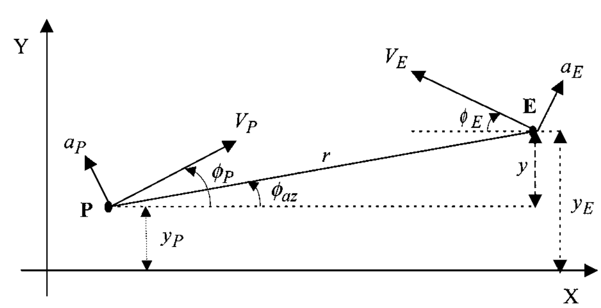

Fig. 1 Engagement geometry.

where

$$x_1 \triangleq y = y_E(t) - y_P(t), \qquad x_2 \triangleq \dot{y}$$
$$x_3 \triangleq a_E, \qquad x_4 \triangleq a_P \tag{5}$$

and the corresponding equations of motion are

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = x_3 - x_4$$
$$\dot{x}_3 = \left(v \cdot a_E^{\max} - x_3\right)/\tau_E + \omega_E, \qquad |v| \le 1$$
$$\dot{x}_4 = \left(u \cdot a_P^{\max} - x_4\right)/\tau_P + \omega_P, \qquad |u| \le 1 \tag{6}$$

where u and v are the controls of the pursuer and evader, respectively. Here, ω_P and ω_E are zero mean Gaussian white process noises with standard deviations σ_P and σ_E, respectively. The problem involves two nondimensional parameters of physical significance: the pursuer/evader maximum maneuverability ratio

$$\mu \triangleq a_P^{\max}/a_E^{\max} \tag{7}$$

and the evader/pursuer time constant ratio

$$\varepsilon = \tau_E/\tau_P \tag{8}$$

Measurements

It is assumed that the interceptor acquires measurements at a given frequency f. The measurements are

$$z_1 = \phi_{az} + v_\phi \cong y/r + v_\phi, \qquad z_2 = a_P + v_P \tag{9}$$

where r, the range between the evader and the pursuer, is measured very accurately relative to the other measurements and is, therefore, assumed to be deterministically known. The angle ϕ_{az} between the current and initial LOS (Fig. 1) is measured with angular noise v_ϕ and the interceptor's own acceleration is measured with noise v_P. These measurement noises are zero mean, white, Gaussian distributed, with standard deviations σ_{ang} and σ_{ap}, respectively.

Lethality Model

A realistic lethality model involving an interceptor missile's warhead and an incoming TBM is very complex. In this study, the probability of target destruction is determined by the following simplified lethality function:

$$P_d(R_k) = \begin{cases} 1, & |x_1(t_f)| \le R_k \\ 0, & |x_1(t_f)| > R_k \end{cases} \tag{10}$$

where R_k is the lethal (kill) radius of the warhead. The interception is successful only if the miss distance $|x_1(t_f)|$ is smaller than the lethal radius of the warhead. A simplifying assumption underlying this model is that the overall reliability of the entire guidance system is 1.

Performance Index

The objective of the interceptor missile is to destroy the incoming TBM with a predetermined probability of success, when it is equipped with a warhead of the smallest possible lethal radius R_k. The required probability of success is measured by the single-shot kill probability (SSKP),[3] defined as

$$\text{SSKP}(R_k) = E\{P_d(R_k)\} \tag{11}$$

where E is the mathematical expectation, taken with respect to the measurement noise and random target maneuver distributions. In this study, the required probability of success is assumed to be 0.95. When the preceding definition is used, the performance index of this game is

$$J = \arg_{R_k}\{\text{SSKP}(R_k) = 0.95\} \tag{12}$$

This performance index is to be minimized by the interceptor (pursuer) and maximized by the evading target.

Maneuver Strategies

Deterministic Analysis

In the ideal, noise-free case, the perfect information assumption forms the worst case from the viewpoint of the defense because it provides the TBM (which is actually blind) with a potential that it does not have. When perfect information is assumed, the maneuvering strategies of both parties can be derived using two different formulations: 1) differential games and 2) optimal control.

Differential Game Solution

The cost function of the perfect information game, to be minimized by the pursuer and maximized by the evader, is the miss distance

$$J = |x_1(t_f)| = |\boldsymbol{D}\boldsymbol{X}(t_f)| \tag{13}$$

where

$$\boldsymbol{D} = (1, 0, 0, 0) \tag{14}$$

In this deterministic case, if the pursuer's warhead lethal radius is larger than the guaranteed miss distance of the game, SSKP = 1 is guaranteed.

The solution of such a game is based on computing the zero effort miss (ZEM), defined as the miss distance that results if both players do not apply any further acceleration commands, and using it as the single state variable of the game. The ZEM can be computed at time t by

$$\text{ZEM}(t) = \boldsymbol{D}\Phi(t_f, t)\boldsymbol{X}(t) \tag{15}$$

where $\Phi(t_f, t)$ is the transition matrix associated with $\dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X}$. In the game with first-order evader dynamics,[12] the ZEM is

$$\text{ZEM}_1 = x_1 + x_2 t_{\text{go}} + \tau_E^2 x_3 \psi(\theta/\varepsilon) - \tau_P^2 x_4 \psi(\theta) \tag{16}$$

where

$$\psi(\zeta) \triangleq e^{-\zeta} + \zeta - 1 \tag{17}$$

In the game where ideal evader dynamics, that is, $\tau_E = 0$, is assumed,[10] the ZEM reduces to

$$\text{ZEM}_0 = x_1 + x_2 t_{\text{go}} - \tau_P^2 x_4 \psi(\theta) \tag{18}$$

Note that the computation of ZEM_0 does not require knowledge of the target acceleration. The optimal strategies in both of these games can be implemented as

$$u^* = \text{sign}[\text{ZEM}_i(t)], \qquad v^* = \text{sign}[\text{ZEM}_i(t)]$$
$$i = 0, 1 \tag{19}$$

The guidance laws based on the optimal pursuer strategies are denoted in the sequel as differential game laws for first-order (DGL/1) and ideal (DGL/0) dynamics, respectively. For most initial conditions of practical importance, the guaranteed miss distance in both game models is constant. For DGL/0, this value is never zero, whereas in DGL/1 it can be nullified if the inequality $\mu\varepsilon \ge 1$ is satisfied.

Remark: In a noise-corrupted scenario, using a bang–bang type strategy might lead to a chattering of the acceleration command. In an actual implementation, to avoid the unwanted chattering of the acceleration command, DGL/0 and DGL/1 can be implemented using a linear strategy (for all initial conditions of practical importance) as was shown in Refs. 10 and 12. In the present study, which concentrates on a comparative evaluation of various estimation techniques (with given guidance laws), this was not deemed necessary.

Optimal Control Solution

Construction of a maneuver strategy based on an optimal control formulation requires an assumption on the future behavior of the opponent. If the guidance law of the interceptor is known, the TBM can perform an optimal evasion strategy.[25–27] In most interceptor missile designs, it has been of common practice to assume a constant target maneuver, resulting in a guidance law denoted as the optimal control guidance law (OGL).[28] In a fashion similar to the preceding derivation, the implementation of this guidance law is based on computing the associated ZEM, which, in this case, is

$$\text{ZEM}_{\text{OGL}} = x_1 + x_2 t_{\text{go}} + 0.5 x_3 t_{\text{go}}^2 - \tau_P^2 x_4 \psi(\theta) \tag{20}$$

The optimal guidance law is

$$u^* = \left(G \big/ a_P^{\max} \tau_P^2\right) \text{ZEM}_{\text{OGL}} \tag{21}$$

where G is the time-varying gain computed as

$$G = \psi(\theta)/(3 + 6\theta - 6\theta^2 + 2\theta^3 - 3e^{-2\theta} - 12\theta e^{-\theta}) \tag{22}$$

This guidance law is based on a linear quadratic formulation assuming unbounded control. As such, it can enforce zero miss distance (SSKP = 1) and minimize the integral of the control effort. Because at interception ($\theta = 0$) the guidance gain becomes infinite, this guidance law saturates in any practical implementation. In this paper, this guidance law is applied, as are all of the other guidance laws, with bounds on the control.

Stochastic Analysis

TBM

In the stochastic (realistic) version of the problem, which includes process and measurement noises, the blind TBM cannot implement a deterministic optimal strategy due to the lack of information. The TBM designer's obvious objective is to avoid interception, in spite of the lack of information, allowing the TBM to hit its designated surface target. Performing no maneuver, or even performing a constant maneuver, generates predictable trajectories, leading to a successful intercept. Thus, the TBM must maneuver randomly. Based on the perfect information game solution, outlined in the preceding section, and on the results presented in Refs. 25–27, the optimal target maneuver sequence has a bang–bang structure (19). Implementation of such a random strategy over the short duration of the endgame consists of a maximal maneuver in one direction, followed by a randomly timed switch to a maximal maneuver in the opposite direction.

Interceptor

Implementation of the perfect information guidance laws presented requires knowledge of the original state variables of the scenario. Unfortunately, they have to be estimated based on the available noisy measurements. As already noted, it has been of common practice to use the estimated states in the deterministic (perfect information) guidance law. Following this practice, the guidance laws (19) and (21) are implemented in this work using the estimated ZEM.

Fast MMAE

In this section an efficient MMAE is presented. A static MMAE is used instead of more advanced MMAE structures, for example, interacting MMAE, because it best fits the underlying assumptions of the TBM interception scenario investigated here. For the sake of comparison, a KF incorporating a SF (a common estimation solution) is also included.

Ordinary MMAE

Based on Magill's pioneering work,[16] the static MMAE can handle a case where the system model is known to be adequately representable by one hypothesis, denoted by α_i, out of a finite set of hypotheses $\{\alpha_i\}_{i=1}^{L}$. The MMAE comprises a set of elemental estimators, each corresponding to a possible hypothesis. In the investigated scenario, the hypotheses are on the TBM maneuver command sequence. In theory, infinite hypotheses are needed for the representation of a bang–bang maneuver command with a randomly timed switch in the endgame scenario. For practical reasons, the duration of the endgame t_f is divided into L time steps of duration Δt_{sw}, resulting in L hypotheses, where the α_i hypothesis corresponds to a switch in the maneuver command at $t_{\text{sw}}^i = i\,\Delta t_{\text{sw}}$, with i ranging from 1 to L.

Let the KF based on the α_i hypothesis be denoted KF(α_i). All of the estimators in the MMAE bank process the same measurements. The output of the ith filter is the estimated vector $\hat{\boldsymbol{X}}_{k/k,\alpha_i}$. The innovations process realization computed by the ith filter is

$$\tilde{z}_{k/\alpha_i} = z_k - \hat{z}_{k/\alpha_i} \tag{23}$$

where k is the discretized time. When the innovations realization of each estimator is used, the a posteriori probability for the correctness of the hypothesis of each estimator can be recursively computed as

$$p(\alpha_i \mid \boldsymbol{Z}_k) = \frac{\exp\left(-\frac{1}{2}\tilde{z}_{k/\alpha_i}^T \Omega_{k/\alpha_i}^{-1} \tilde{z}_{k/\alpha_i}\right)}{c|\Omega_{k/\alpha_i}|^{\frac{1}{2}}} p(\alpha_i \mid \boldsymbol{Z}_{k-1}) \tag{24}$$

where Ω_{k/α_i} is the innovations covariance matrix and c is a normalization coefficient, which is computed to satisfy

$$\sum_{i=1}^{L} p(\alpha_i \mid \boldsymbol{Z}_k) = 1 \tag{25}$$

The total scheme state estimate can now be computed using one of the following approaches: 1) MMSE, probability weighted average of the estimated vectors from all elemental filters, based on their a posteriori probabilities, or 2) MAP, the state estimate of the total scheme is taken from the filter associated with the maximum a posteriori probability.

The MMAE algorithm is shown in Fig. 2.

Efficient Algorithm

In principle, the implementation of an MMAE requires the use of as many KFs as the number of hypotheses. In the investigated scenario, where in theory infinite models are needed, increasing the number of models in the bank improves the homing performance (up to a certain limit), as will be shown in the sequel. Because the computational effort increases linearly with the number of KFs, the number of models in the bank is usually constrained by the available computational power. However, as will be shown next, this constraint can be significantly relaxed in the special case under consideration.

The idea underlying the fast MMAE is based on that, in the case under investigation, the hypotheses are on the TBM maneuver command sequence. Thus, the various filters that compose the MMAE bank differ just in their hypothesis on the target maneuver switch time. The important observations, leading to a substantial computational load saving, are the following. First, at any time t during the scenario, all filters KF(α_i) in the MMAE bank whose underlying hypotheses correspond to target maneuver switch times that belong to the future, that is, $t_{\text{sw}}^i > t$, can be represented by a single filter. Second, at time t the MMAE scheme need not contain a filter KF(α_j) whose underlying hypothesis is that the target has already performed a maneuver switch, that is, $t_{\text{sw}}^j \ll t$, if that filter has not already been found by the MMAE scheme to be correct. Incorporating these observations within the MMAE structure results in a highly efficient algorithm that comprises the following three subprocesses.

Elemental Filter Aggregation

All filters in the bank that do not differ in their underlying assumptions at the current time are aggregated, that is, those filters that are based on models assuming a maneuver switch that has not yet occurred are all represented by a single filter because their state estimates, estimation error covariances, and a posteriori probabilities are identical. Thus, the number of filters represented by the aggregated filter at time t

$$L_{\text{ag}}(t) \triangleq L - j(t) \tag{26}$$

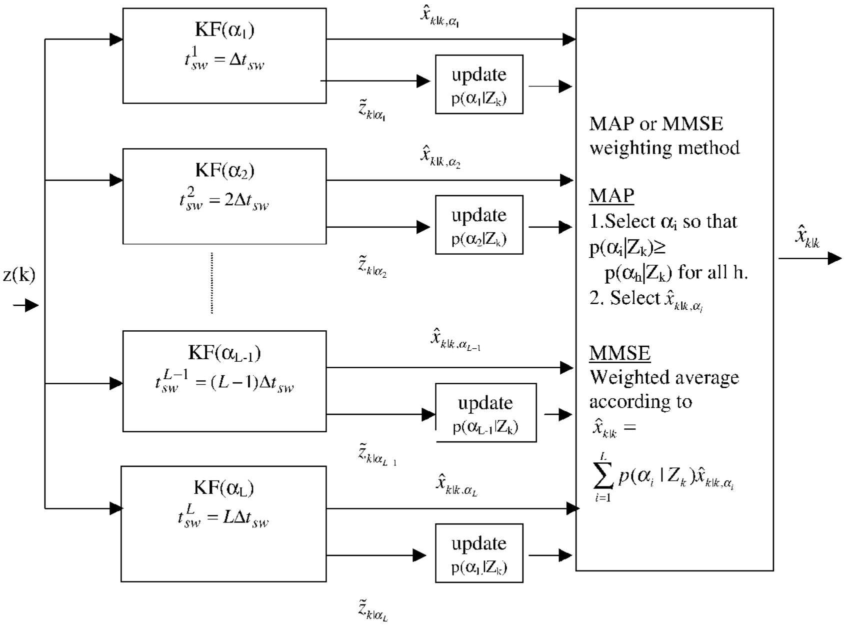

Fig. 2 Ordinary MMAE algorithm.

decreases monotonically with $j(t)$, where

$$j(t) = \mathrm{int}\left(t^- / \Delta t_{\mathrm{sw}}\right) \tag{27}$$

is the number of models in which switches have occurred before time t.

When the current time t reaches the switching time of the ith model, a new filter, corresponding to this model, is initialized. The new filter is assigned the same state estimate and estimation error covariance matrix as the representative (aggregated) estimator at that time. Assuming that the aggregated filter represents $L_{\mathrm{ag}}(t)$ identical models, the new filter is assigned the a posteriori probability

$$p(\alpha_i \mid \boldsymbol{Z}_k) = [1/L_{\mathrm{ag}}(t)]p(\alpha_{\mathrm{ag}} \mid \boldsymbol{Z}_k) \tag{28}$$

where $p(\alpha_{\mathrm{ag}} \mid \boldsymbol{Z}_k)$ is the a posteriori probability of the aggregated filter before the initialization of the new filter. Obviously, after initializing the new filter, its probability is subtracted from that of the aggregated filter.

MMAE Filter Pruning

Whenever a maneuver command switch occurs, it is assumed that the MMAE bank identifies it within a time interval T_{id}, computed as

$$T_{\mathrm{id}} = s_f \cdot T_{\mathrm{id}}^{\min} \tag{29}$$

where s_f is an algorithm tuning parameter (safety factor), chosen by the estimator designer, and $T_{\mathrm{id}}^{\min}$ is the minimum time required for maneuver command detection. It can be shown that a target maneuver command can be detected, with false alarm probability of about 0.05, when the absolute value of the measured deviation of the target position from its nominal trajectory exceeds twice the value of the standard deviation of the respective measurement noise.[29] The position deviation of a missile, with first-order dynamics, due to a lateral acceleration step command of magnitude a_E^c at $t = 0$ can be approximated by

$$\Delta y(t) \cong a_E^c t^3 / 6\tau_E \tag{30}$$

Hence, the minimal detection time can be approximated by

$$T_{\mathrm{id}}^{\min} \cong \arg_t\{\Delta y(t) = 2\sigma_y = 2r\sigma_{ang}\} \cong \sqrt[3]{12\tau_E r\sigma_{ang} / a_E^c} \tag{31}$$

where σ_y is the standard deviation of the measurement noise of $y(t)$.

If a certain maneuver switch is not identified by its corresponding filter (that is, the a posteriori probability of the corresponding filter stays below a predetermined threshold, chosen as 0.8 in this study) within the time interval T_{id} after the expected maneuver switch, then it is reasonable to assume that it will also not be identified in the future as being the correct model. Therefore, this filter can be discarded.

Discarding an old filter takes place simultaneously with the initialization of a new filter (described earlier). For a maneuver detection time T_{id}, the number of models about which a decision cannot yet be made at any time is $\mathrm{int}(T_{\mathrm{id}}/\Delta t_{\mathrm{sw}})$. Including the aggregated filter, the total number of filters in the efficient MMAE bank is

$$L_s = \mathrm{int}(T_{\mathrm{id}}/\Delta t_{\mathrm{sw}}) + 1 \tag{32}$$

The total number of models that have to be used, for a given scenario and a required resolution, is

$$L = t_f / \Delta t_{\mathrm{sw}} \tag{33}$$

Therefore, the size of the filter bank is reduced by

$$L_s / L \cong T_{\mathrm{id}} / t_f \tag{34}$$

Thus, the detection time interval and the scenario duration determine a physical bound on the achievable filter bank size reduction.

Unified Covariance Computation

In the case under study, it is possible to further reduce the algorithm's computational load by computing the covariance and gain matrices only once for all L_s KFs in the MMAE bank, as was done in Ref. 18, which dealt with a different scenario. This is based on the observation that in the investigated scenario the hypotheses relate only to the evader's control; hence, the transition matrix of the homogeneous system is identical for all of the estimators in the bank, resulting in identical covariance and gain matrices for all KFs.

The well-known discrete-time KF equations may be found in any estimation textbook, for example, Ref. 30, and, hence, they are not repeated here.

The fast MMAE algorithm is shown in Fig. 3.

SF

The SF method allows for the representation of a system driven by an arbitrary (not necessarily white) stochastic input by an augmented system that is excited by white noise only. This greatly simplifies the design of the estimator. The augmented system, which

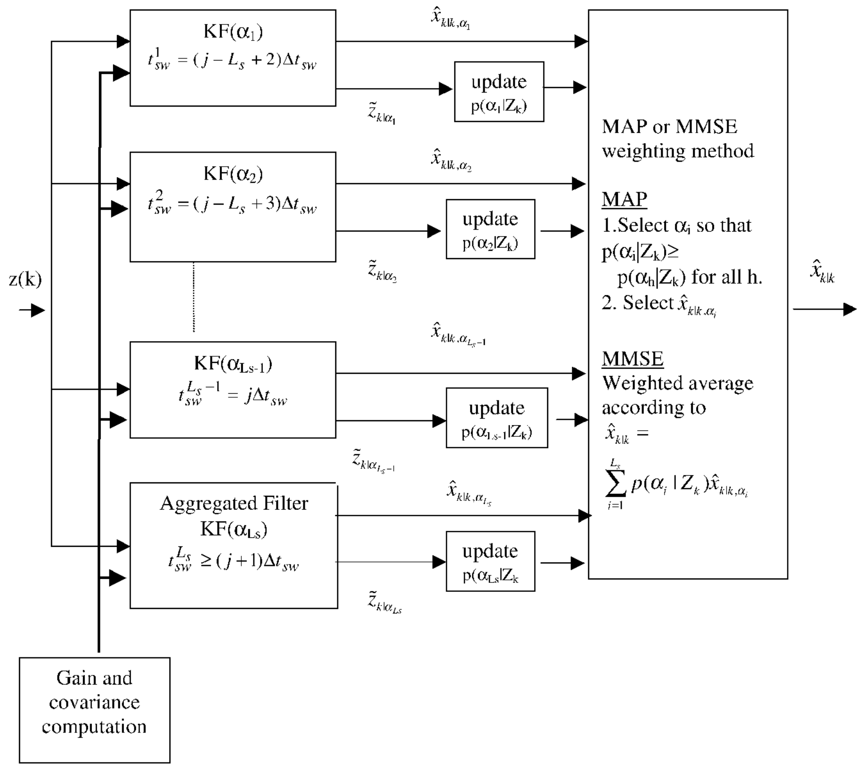

Fig. 3 Fast MMAE algorithm.

comprises the original system and the SF, is designed such that its output possesses the same first- and second-order statistical moments as the output of the original system. The equivalent SF for a randomly timed maneuver[13] is an integrator excited by white noise with power spectral density of $4 \cdot (a_E^{\max})^2/t_f$. This makes it necessary to augment the system by one state variable corresponding to the evader's maneuver command.

Computational Load

The computational load associated with the various filtering algorithms considered in this paper has been estimated in Ref. 31 by counting floating-point operations (FLOPs), where each basic algebraic operation was counted as one FLOP (complying with MATLAB®'s convention). For brevity, only the final computational load formulas, which can be used as design formulas, are given here:

$$\text{load} = \begin{cases} t_f f\big[L_{e1}(3n^3+2m^3+5n^2m+5nm^2-1.5n^2-3nm) \\ \quad +L_{e2}(2m^2+4mn+2m+5)\big]+(t_f/\Delta t)\big[L_{e1}n(3n^2 \\ \quad -1.5n+2s^2+sn-s)+L_{e2}n(2n+4)\big], \quad \text{MMAE} \\ t_f f(3n^3+2m^3+5n^2m+5m^2n-1.5n^2+mn) \\ \quad +(t_f/\Delta t)(3n^3+0.5n^2+2s^2+ns+2n-s), \quad \text{KF/SF} \end{cases} \tag{35}$$

where

$$L_{e1} = \begin{cases} 1, & \text{fast MMAE} \\ L, & \text{MMAE} \end{cases} \tag{36}$$

$$L_{e2} = \begin{cases} L_s, & \text{fast MMAE} \\ L, & \text{MMAE} \end{cases} \tag{37}$$

$$n = \dim(\boldsymbol{X}), \qquad m = \dim(\boldsymbol{Z}), \qquad s = \dim(\boldsymbol{W}) \tag{38}$$

where $\boldsymbol{W}$ is the process noise vector.

The reduction in the computational load for the investigated scenario when using the fast MMAE instead of the conventional one is presented in Fig. 4, along with a comparison to the KF/SF. Note that the computational load associated with the proposed fast algorithm is reduced by about an order of magnitude compared with the ordinary MMAE, becoming comparable to that of a KF/SF. Viewed from a different perspective, it can be stated that for a given, limited, computational power, using the proposed fast MMAE allows for the incorporation of many more models in the bank, that is, it enables the representation of the hypothesis space with a much higher resolution. This obviously leads to superior homing accuracy, as will be shown in the sequel.

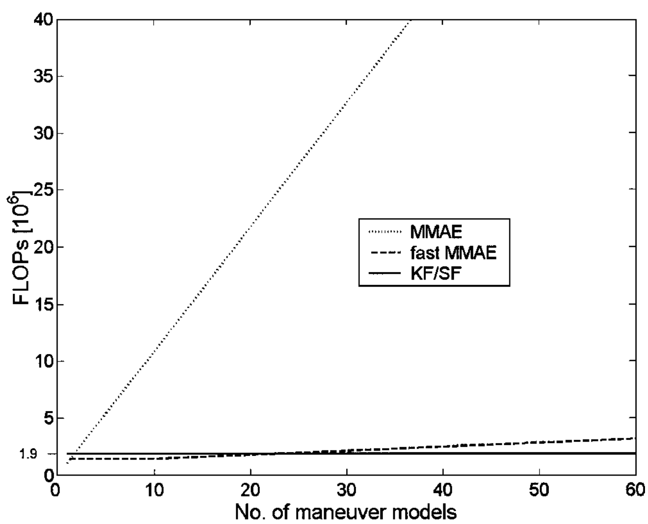

Fig. 4 Comparison of the computational load for MMAE (ordinary and fast) and KF/SF.

Simulation Study

When the implementation of various guidance laws was simulated with the two different estimation approaches, sets of 100 Monte Carlo simulation runs with independent random switching times and noise samples were used. To allow for a comparison between the various investigated cases, the same random number generator seed has been selected.

The parameters of the discrete simulation (with time step $\Delta t = 0.001$ s) are summarized in Table 1, where g stands for gravity acceleration. They were chosen to satisfy the inequality $\mu\varepsilon \geq 1$, for which, in a perfect information engagement, zero miss distance is guaranteed when using DGL/1.

Estimation Performance

In the investigated scenario, a resolution of $\Delta t_{\rm sw} = 0.1$ s was chosen; thus, for a 3.0-s endgame duration, the MMAE bank consists of 30 models, corresponding to maneuver switches at $t = 0.1i$ s, with i ranging from 1 to 30. At maximum range, $T_{\rm id}$ ($r = 16.5$ km) $\cong 0.68$ s; hence, using Eq. (32), 7 elemental filters are required to run concurrently in the efficient MMAE bank instead of 30 in the full-order MMAE bank.

In Figs. 5 and 6 the same example ($t_{\rm sw} = 1.52$ s, which occurs at $r \cong 8$ km) is used for estimation performance comparison between the efficient MMAE using the MMSE or MAP methods and the KF/SF. Figure 5 shows the a posteriori probability of each of the efficient MMAE filters. Note that the aggregated filter, corresponding to the assumption that no maneuver has yet occurred, has a high probability of about 0.8 before the actual switch. After the switch has occurred, it takes about 0.4 s for the MMAE to detect the existence of the maneuver and identify it (converging to the model with the closest switch, $t_{\rm sw} = 1.5$ s), which agrees well with the precomputed approximate value of $T_{\rm id}^{\rm min}$ ($r \cong 8$ km) $\cong 0.45$ s. Notice that every 0.1 s, when a new filter is initialized and an old filter is discarded, the filters assigned to the various hypotheses change their indices in the MMAE bank, for example, at $t = 2$ s, filter 6 replaces filter 5 in the bank and is assigned filter 5's previous probability of 0.95. In Fig. 6 the estimation of the evader's acceleration is shown for the various estimation methods. It is clear that, only after the MMAE identifies the correct model, the estimated states of the MMAE converge to the true states. It can also be seen that using the MAP criterion creates (before convergence to the correct model) jumps in the estimated states, due to switches between the models. It is apparent that the estimation errors of the MMAE/MMSE scheme are the smallest of the three methods.

Table 1 Simulation parameters

Evader	Pursuer	Scenario
$a_E^{\max} = 21.5\ g$	$a_P^{\max} = 48.4\ g$	$\mu = 2.25$
$\tau_E = 0.2$ s	$\tau_P = 0.2$ s	$\varepsilon = 1$
$V_E = 2.7$ km/s	$V_P = 2.8$ km/s	$X_0 = 16.5$ km
$\phi_E(0) = 0$ deg	$\phi_P(0) = 0$ deg	$t_f = 3$ s
$\sigma_E = 1\ g$	$\sigma_P = 0.1\ g$	$f = 200$ Hz
$\sigma_{ang} = 1$ mrad	$\sigma_{ap} = 0.1\ g$	$s_f = 1.2$

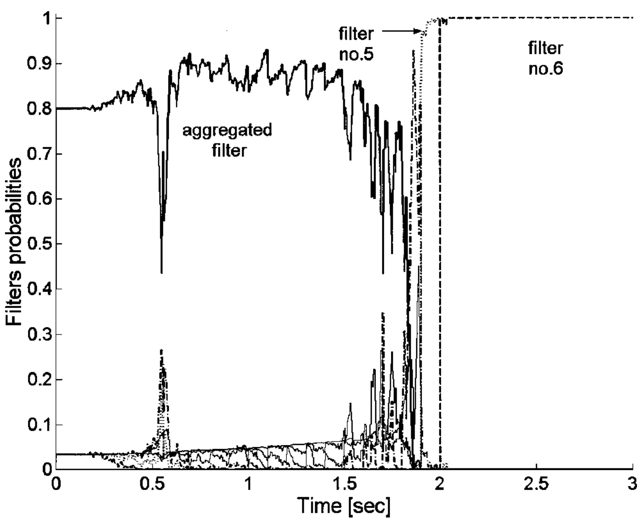

Fig. 5 Typical performance of the fast MMAE: a posteriori probabilities of elemental filters; $L = 30$, $L_s = 7$, and $t_{\rm sw} = 1.52$ s.

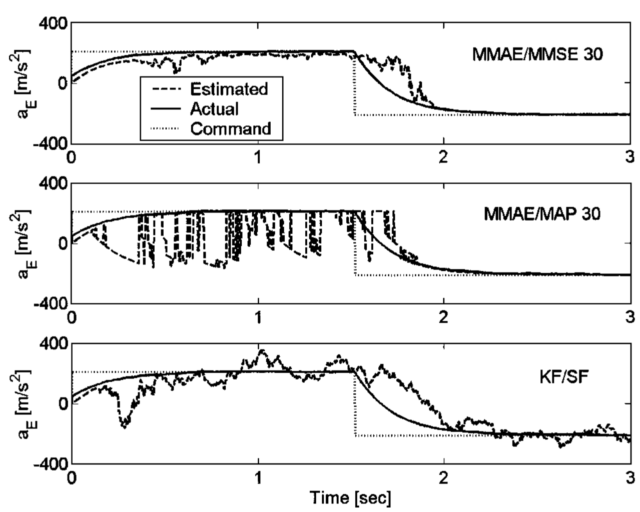

Fig. 6 Typical closed-loop estimation performance of MMAE (with MMSE and MAP fusion methods) and KF/SF; guidance law is DGL/0, MMAE parameters are $L = 30$ and $L_s = 7$ with target maneuver switch time $t_{\rm sw} = 1.52$ s.

Homing Performance

Figures 7, 8, and 9 present the homing performance of the various guidance laws when combined with the estimation approaches MMAE/MMSE, MMAE/MAP, and KF/SF, respectively.

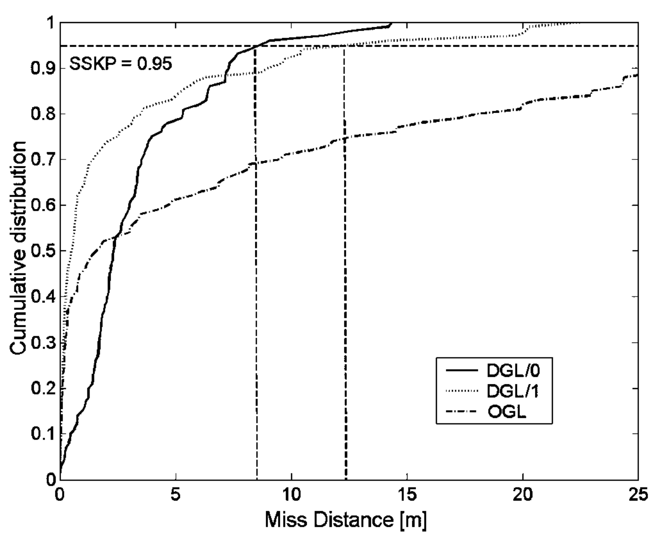

Fig. 7 Homing performance of DGL/0, DGL/1, and OGL with fast MMAE/MMSE, $L = 30$.

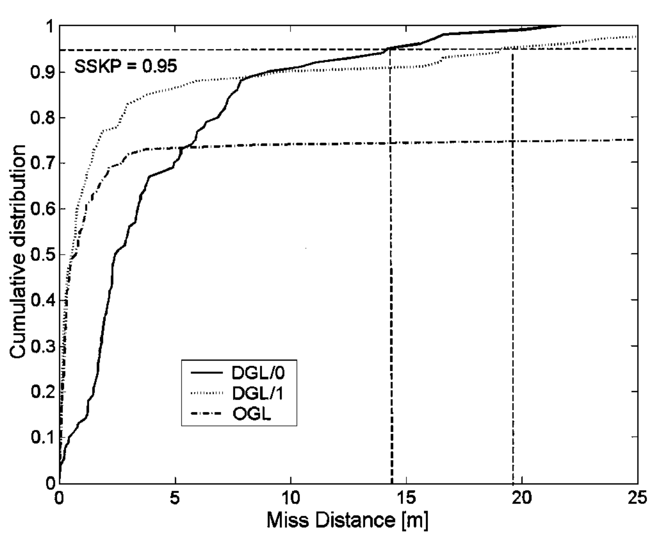

Fig. 8 Homing performance of DGL/0, DGL/1, and OGL with fast MMAE/MAP, $L = 30$.

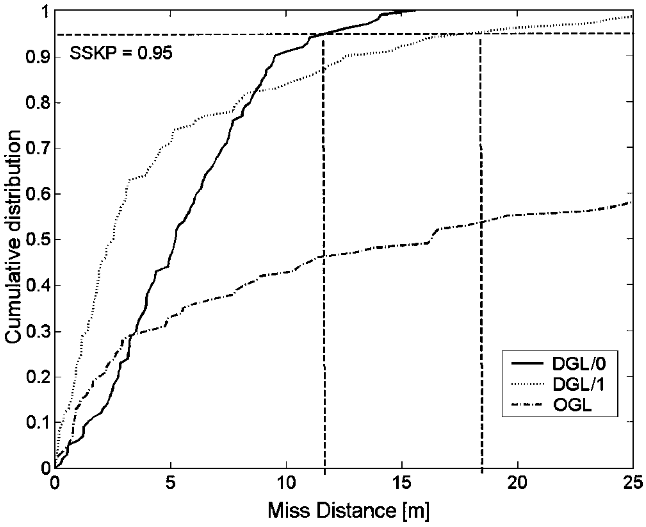

Fig. 9 Homing performance of DGL/0, DGL/1, and OGL using a KF/SF.

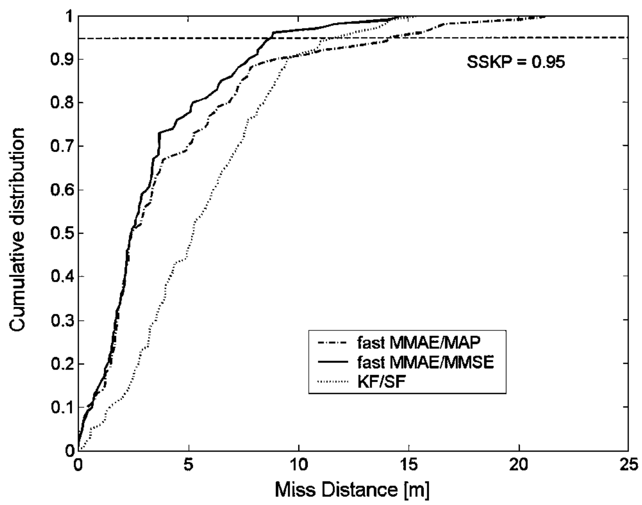

Fig. 10 Homing performance of DGL/0 with fast MMAE (with MMSE and MAP fusion methods) and KF/SF.

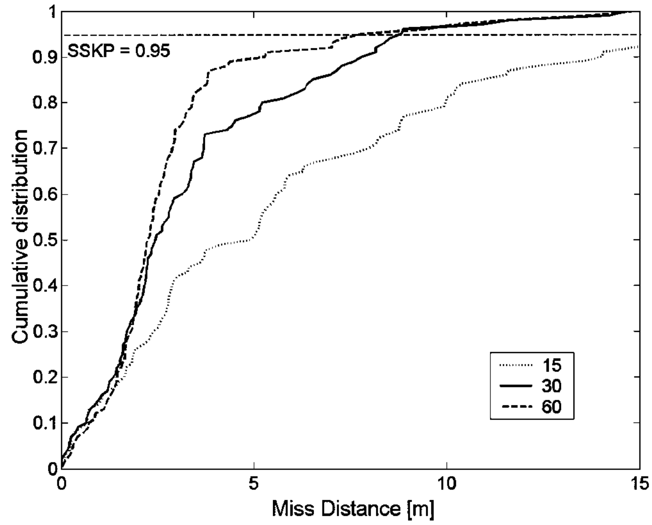

Fig. 11 Homing performance of DGL/0 vs hypothesis space resolution; fast MMAE/MMSE used.

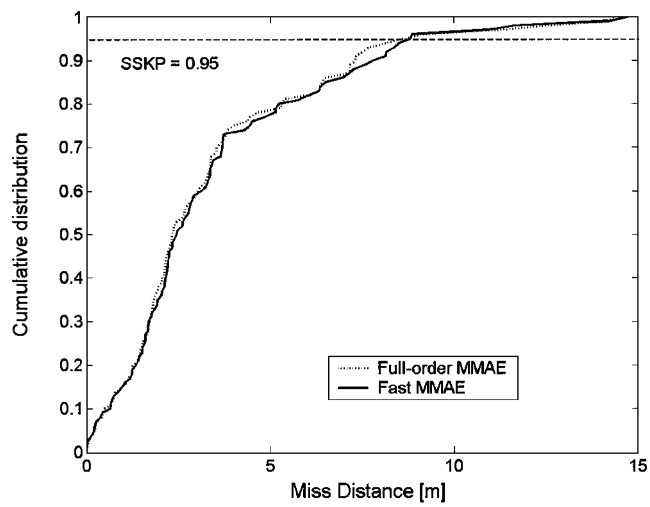

Fig. 12 Effect of model reduction on DGL/0 homing performance.

It is apparent that, although the use of OGL yields, in many cases, negligible miss distances (between 15–65%, depending on the kind of estimator used), in other cases it yields very large values (maximum miss distances between 35 and 140 m). On the other hand, the use of DGL/1 yields small miss distances with a higher probability, and the nonnegligible miss distances are much smaller than those obtained using OGL. A remarkable conclusion, becoming evident on observing Figs. 7–9, is that in this example DGL/0 is better than DGL/1, the optimal guidance law for the perfect information game. This conclusion is based on noting that the maximum miss distances associated with DGL/0, as well as the probability for large miss distances, for example, larger than 10 m, associated with this law, are the smallest. Viewed from a different perspective, it can be stated that to achieve a given SSKP, for example, SSKP = 0.95, the use of DGL/0 requires a warhead with the smallest lethal radius for all of the three estimation methods investigated. This superiority of DGL/0 over DGL/1 can be viewed as a numerical validation of the assertion that, in realistic, noise-corrupted BMD scenarios, CEP does not hold.

Based on these results, DGL/0 was chosen for the homing performance comparison presented in Figs. 10 and 11. In Fig. 10 the MMAE/MMSE, MMAE/MAP, and KF/SF estimators are compared. It is apparent that an MMAE/MMSE with a resolution of 30 provides the best homing performance. Figure 11 presents the homing performance vs the resolution of the MMAE/MMSE bank. As can be seen, increasing the resolution from 15 to 30 leads to a substantial performance improvement in more than 70% of the cases. Increasing the resolution further, from 30 to 60, has a smaller effect on a smaller portion (about 50%) of the cases.

In passing, note that reducing the number of concurrently active filters in the efficient algorithm does not incur substantial performance degradation. This is demonstrated in Fig. 12, which shows the performance of DGL/0 combined with both ordinary and fast MMAE/MMSE. Similar results have been obtained for the other guidance laws (DGL/1 and OGL) and estimation method (MMAE/MAP) investigated in this work.

Conclusions

The work described demonstrates that, in a future BMD scenario, real-time application of a fast, computationally efficient MMAE scheme is feasible. It has been shown that, by using the proposed novel efficient MMAE algorithm, a dramatic reduction in the computational load can be achieved, facilitating a substantial increase in the resolution of the discretized hypothesis space. This, in turn, leads to improved estimation performance, and, as a consequence, to better guidance accuracy.

The reduction in the computational load is achieved mainly by noting that, when the MMAE hypotheses relate only to the timing of the evader's control, only a few elemental filters are needed for proper representation of a high-resolution hypothesis space. This observation leads to the introduction of the aggregation and pruning operators, which enable the use of a greatly reduced MMAE bank, compared with the large, ordinary MMAE of the same accuracy that would have been used had these operations not been employed. In addition, in the scenarios of interest, the transition matrix (of the homogeneous system) is identical for all estimators in the bank. Therefore, the covariance and gain matrices need be computed only once per filtering cycle.

Applying this efficient method, which employs a computational load reduction scheme different from other methods outlined in earlier papers, circumvents the main drawback of the MMAE, which made its use impractical for real-time implementations in most interception scenarios. For a given computational capacity, this efficient MMAE allows for the inclusion of many more target evasion models in the estimator bank, leading to a significant improvement in homing performance. Note that the efficient MMAE can be implemented not only for bang–bang maneuvers but for all target maneuver command models with different initialization timing.

Based on an extensive Monte Carlo simulation study, an evaluation of the homing performance of an interceptor guidance system processing noisy measurements using the fast MMAE has been carried out. Two MMAE weighting methods, namely, MAP and MMSE, have been compared along with a conventional KF incorporating an SF. Coupled with guidance laws derived using differential game and optimal control theories, the MMSE weighting method was found to provide the best homing performance if a sufficient number of models is used in the MMAE bank.

It was also found that, in this scenario, with the assumed relatively high noise level and the given SSKP criterion, DGL/0 provides the best homing performance. The reason is that this guidance law does not use information on the target acceleration; hence, it is less susceptible to the acceleration estimation error. Nonetheless, it is still of prime importance to model correctly this acceleration for the sake of the estimator performance. Inaccurate modeling will result in degraded estimation and, consequently, in poor homing performance.

The superiority of DGL/0 demonstrates that, when certainty equivalence cannot be assumed, one should not automatically use the optimal deterministic guidance law (in this case DGL/1, which guarantees zero miss distance in the perfect information case). The construction of a new guidance law that fully takes into account the estimation characteristics is, currently, an open problem. Such a guidance law should be of prime interest to the guided missile community.

The results of this study indicate that using the proposed efficient MMAE with an appropriate guidance law will facilitate successful interception of highly maneuvering TBMs expected in the future.

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